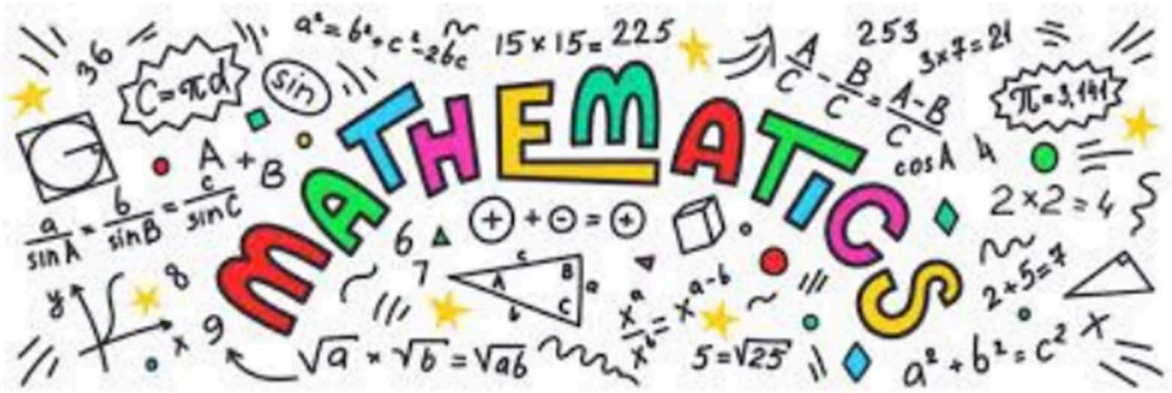
SAGEMONT

PREPARATORY SCHOOL

Summer Mathematics Packet for Students entering AP Calculus



Complete ALL questions, showing ALL work neatly on A4 size paper

Algebra and Polynomial Functions

1. Simplify: $(x^3 - 2x^2 + 4x) - (x^3 - 3x^2 + 5x)$.
2. Factor the quadratic expression: $x^2 - 6x + 8$.
3. Solve the equation: $3x^2 + 5x - 2 = 0$.
4. Find the roots of the polynomial $x^3 - 4x^2 + x - 4 = 0$.
5. Perform synthetic division to divide $x^3 - 3x^2 + 2x - 6$ by $x - 2$.
6. Find the x-intercepts of $f(x) = x^2 - 9$.
7. Factor completely: $2x^2 - 8x$.
8. Solve for x: $x^4 = 16$.
9. Find the quotient and remainder when $x^3 - 5x + 6$ is divided by $x - 2$.
10. Determine the end behavior of the polynomial function $f(x) = -2x^3 + 5x^2 - x + 1$.

Rational Expressions and Functions

11. Simplify: $\frac{2x^2 - 3x}{x^2 - 4}$.
12. Solve for x: $\frac{3x}{x-1} = 6$.
13. Find the domain of $f(x) = \frac{5}{x^2 - 9}$.
14. Simplify the expression: $\frac{x^2 + 3x + 2}{x^2 + 5x + 6}$.
15. Solve for x: $\frac{1}{x+2} - \frac{2}{x-3} = 0$.
16. Determine the vertical asymptotes of $f(x) = \frac{x+3}{x^2 - 4}$.
17. Find the horizontal asymptote of $f(x) = \frac{3x^2 + 5x - 2}{2x^2 + x - 4}$.
18. Factor the denominator: $x^2 - 1$.
19. Solve for x: $\frac{x+3}{x-2} = 4$.
20. Perform the division: $\frac{x^2 - 4x + 4}{x - 2}$.

Exponents and Logarithms

21. Simplify: $(3x^2)^3$.
22. Solve for x: $2^{x+2} = 32$.
23. Solve for x: $\log(x + 1) = 3$.
24. Expand the expression: $\log_3(x^2)$.
25. Express $\log_5(125)$ as an exponent.
26. Simplify: $\log_2 64$.
27. Solve the logarithmic equation: $\log(x) + \log(x - 3) = 1$.
28. Solve for x: $5^{x-1} = 25$.
29. Convert $\log_{10} 1000$ into exponential form.
30. Simplify: $2^x \cdot 2^{x+1}$.

Functions and Graphs

31. Find the domain and range of $f(x) = \sqrt{x - 3}$.
32. Graph the function $f(x) = |x - 2| + 3$.
33. Describe the transformation of $f(x) = \sin(x)$ to $f(x) = 2\sin(x) + 1$.
34. Find the equation of a line that passes through the points $(2, 3)$ and $(4, 7)$.
35. Determine the vertex of the quadratic function $f(x) = x^2 - 4x + 5$.

36. Find the x- and y-intercepts of $f(x) = x^2 + 2x - 8$.
37. Sketch the graph of $y = -x^2 + 4x - 3$.
38. Identify the asymptotes of the rational function $f(x) = \frac{3x+5}{x^2-1}$.
39. Find the inverse of the function $f(x) = 3x - 4$.
40. Find the zeros of the function $f(x) = x^3 - 3x^2 + 2x$.

Trigonometry

41. Solve for x in the equation $\sin(x) = \frac{1}{2}$, where $0 \leq x \leq 2\pi$.
42. Find the amplitude, period, and phase shift of $y = 3\cos(2x - \pi)$.
43. Solve for x in the equation $\cos(x) = 0$, where $0 \leq x \leq 2\pi$.
44. Verify the identity: $\sin^2(x) + \cos^2(x) = 1$.
45. Simplify: $\sin(x) \cdot \cos(x)$ using a double-angle identity.
46. Convert 240° to radians.
47. Find the exact value of $\sin(45^\circ)$ and $\cos(45^\circ)$.
48. Use the unit circle to find the value of $\sin\left(\frac{7\pi}{4}\right)$.
49. Solve for x in the equation $\tan(x) = \sqrt{3}$, where $0 \leq x \leq 2\pi$.
50. Find the period of the function $y = 2\sin(3x)$.