To: Sagemont School students entering a precalculus course in August, 2019

From: The Sagemont School, Upper School, Math Department

Entering Precalculus Summer Packet

The math department strongly recommends that all students-entering-calculus review foundational algebra/geometry skills throughout the Summer of 2019.

Please find attached an algebra/geometry review, found on the Internet, developed by Penncrest High School. You may find and use a different review if you prefer, such as a thorough SAT Math review.

Please save all of your handwork. In other words, when precalculus starts in August, your precalculus teacher does not want to see a list of answers with no work shown. Rather, your precalculus teacher wants to see your hand work for all not-easily-done-in-your-head problems.

If you have further questions, please e-mail Mr. Carlson at

dcarlson@sagemont.com

Best wishes!

Radicals:

To simplify means that 1) no radicand has a perfect square factor <u>and</u>
2) there is no radical in the denominator (rationalize).

Recall – the **Product Property** $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ and the **Quotient Property** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Examples: Simplify $\sqrt{24} = \sqrt{4} \cdot \sqrt{6}$ find a perfect square factor

$= 2\sqrt{6}$ simplify

Simplify $\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ split apart, then multiply by both the numerator and the denominator by $\sqrt{2}$

$= \frac{\sqrt{14}}{\sqrt{4}} = \frac{\sqrt{14}}{2}$ multiply straight across and simplify

If the denominator contains 2 terms –
multiply the numerator and the denominator by the ***conjugate*** of the denominator
The ***conjugate*** of $3+\sqrt{2}$ is $3-\sqrt{2}$ (the sign changes between the terms).

Simplify each of the following.

1. $\sqrt{32}$
2. $\sqrt{(2x)^8}$
3. $\sqrt[3]{-64}$
4. $\sqrt{49m^2n^8}$
5. $\sqrt{\frac{11}{9}}$
6. $\sqrt{60} \bullet \sqrt{105}$
7. $(\sqrt{5}-\sqrt{6})(\sqrt{5}+\sqrt{2})$

Rationalize.

8. $\frac{1}{\sqrt{2}}$
9. $\frac{2}{\sqrt{3}}$
10. $\frac{3}{2-\sqrt{5}}$

Complex Numbers:

Form of complex number - $a+bi$

Where a is the "real" is part and bi is the "imaginary" part

Always make these substitutions $\sqrt{-1}=i$ and $i^2=-1$

- To simplify: pull out the $\sqrt{-1}$ before performing any operation

Example: $\sqrt{-5}=\sqrt{-1}\cdot\sqrt{5}$ Pull out $\sqrt{-1}$
$= i\sqrt{5}$ Make substitution

Example: $(i\sqrt{5})^2 = i\sqrt{5}\cdot i\sqrt{5}$ List twice
$= i^2\sqrt{25}$ Simplify
$= (-1)(5) = -5$ Substitute

- Treat i like any other variable when +, -, ×, or ÷ (but always simplify $i^2=-1$)

Example: $2i(3+i)=2(3i)+2i(i)$ Distribute
$= 6i+2i^2$ Simplify
$= 6i+2(-1)$ Make substitution
$= -2+6i$ Simplify and rewrite in complex form

- Since $i=\sqrt{-1}$, no answer can have an 'i' in the denominator **RATIONALIZE!!**

Simplify.

9. $\sqrt{-49}$

10. $6\sqrt{-12}$

11. $-6(2-8i)+3(5+7i)$

12. $(3-4i)^2$

13. $(6-4i)(6+4i)$

Rationalize.

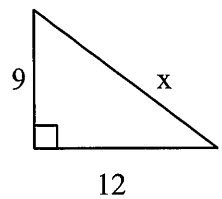
14. $\frac{1+6i}{5i}$

Geometry:

Pythagorean Theorem (right triangles): $a^2 + b^2 = c^2$

Find the value of x.

15.



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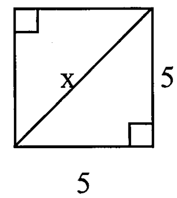

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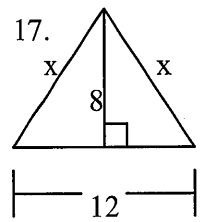
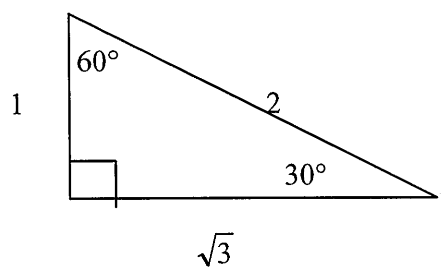
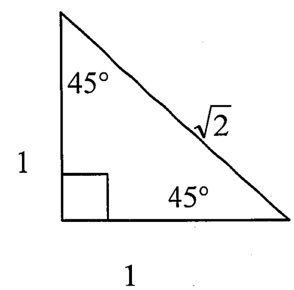


18. A square has perimeter 12 cm. Find the length of the diagonal.

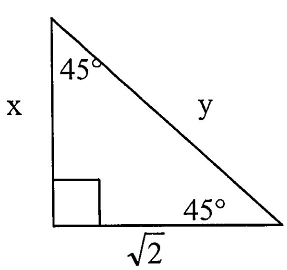
* In $30° - 60° - 90°$ triangles, sides are in proportion $1, \sqrt{3}, 2$.	*In $45° - 45° - 90°$ triangles, sides are in proportion $1, 1, \sqrt{2}$.





Solve for x and y.

19.



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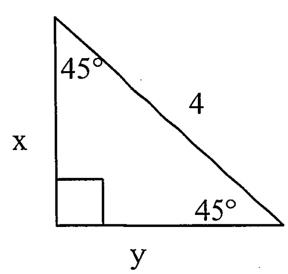

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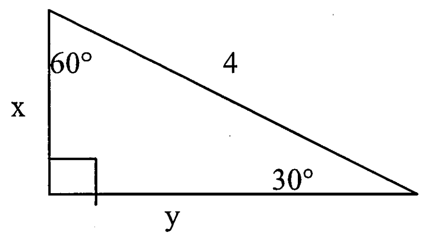

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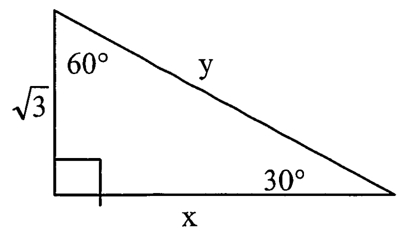


Equations of Lines:

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: y = c (slope is 0)

Standard Form: $Ax + By = C$

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

23. State the slope and y-intercept of the linear equation: 5x – 4y = 8.

24. Find the x-intercept and y-intercept of the equation: 2x – y = 5

25. Write the equation in standard form: y = 7x – 5

Write the equation of the line in slope-intercept form with the following conditions:

26. slope = -5 and passes through the point (-3, -8)

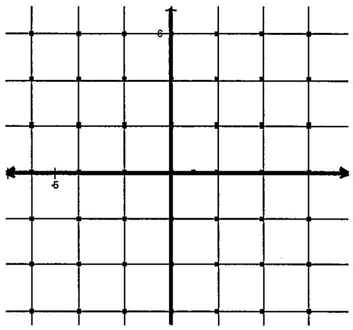
27. passes through the points (4, 3) and (7, -2)

28. x-intercept = 3 and y-intercept = 2

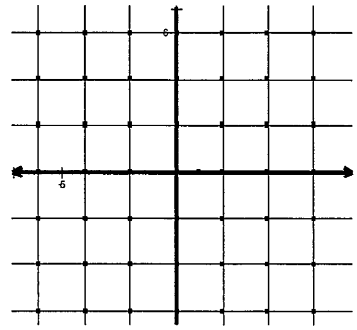
Graphing:

Graph each function, inequality, and / or system.

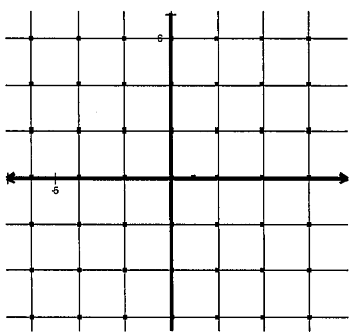
29. $3x-4y=12$



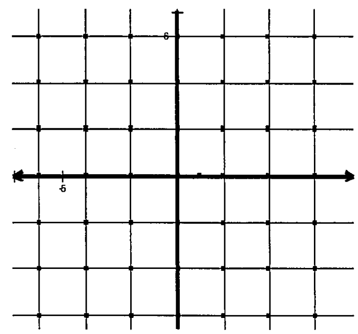
30. $\begin{cases} 2x+y=4 \\ x-y=2 \end{cases}$



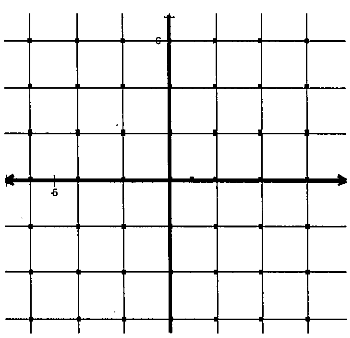
31. $y<-4x-2$



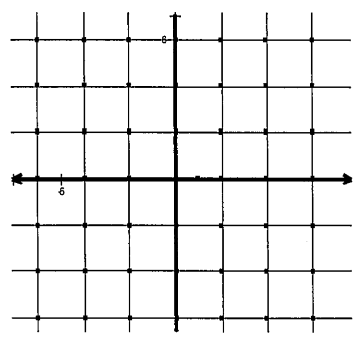
32. $y+2=|x+1|$



33. $y>|x|-1$



34. $y+4=(x-1)^2$



Vertex: ____________________

x-intercept(s): ____________________

y-intercept(s): ____________________

Systems of Equations:

$3x + y = 6$

$2x - 2y = 4$

Substitution:
Solve 1 equation for 1 variable.
Rearrange.
Plug into 2nd equation.
Solve for the other variable.

Elimination:
Find opposite coefficients for 1 variable.
Multiply equation(s) by constant(s).
Add equations together (lose 1 variable).
Solve for variable.

Then plug answer back into an original equation to solve for the 2nd variable.

$y = 6 - 3x$	solve 1st equation for y
$2x - 2(6 - 3x) = 4$	plug into 2nd equation
$2x - 12 + 6x = 4$	distribute
$8x = 16$	simplify
$x = 2$	

$6x + 2y = 12$	multiply 1st equation by 2
$2x - 2y = 4$	coefficients of y are opposite
$8x = 16$	add
$x = 2$	simplify

Plug $x = 2$ back into original

$3(2) + y = 6$

$6 + y = 6$

$y = 0$

Solve each system of equations. Use any method.

35. $\begin{cases} 2x + y = 4 \\ 3x + 2y = 1 \end{cases}$

36. $\begin{cases} 2x + y = 4 \\ 3x - y = 14 \end{cases}$

37. $\begin{cases} 2w - 5z = 13 \\ 6w + 3z = 10 \end{cases}$

Exponents:

TWO RULES OF ONE

1. $a^1 = a$

Any number raised to the power of one equals itself.

2. $1^a = 1$

One to any power is one.

ZERO RULE

3. $a^0 = 1$

Any nonzero number raised to the power of zero is one.

PRODUCT RULE

4. $a^m \cdot a^n = a^{m+n}$

When multiplying two powers that have the same base, add the exponents.

QUOTIENT RULE

5. $\dfrac{a^m}{a^n} = a^{m-n}$

When dividing two powers with the same base, subtract the exponents.

POWER RULE

6. $(a^m)^n = a^{m \cdot n}$

When a power is raised to another power, multiply the exponents.

NEGATIVE EXPONENTS

7. $a^{-n} = \dfrac{1}{a^n}$ and $\dfrac{1}{a^{-n}} = a^n$

Any nonzero number raised to a negative power equals its reciprocal raised to the opposite positive power.

Express each of the following in simplest form. Answers should not have any negative exponents.

38. $5a^0$

39. $\dfrac{3c}{c^{-1}}$

40. $\dfrac{2ef^{-1}}{e^{-1}}$

41. $\dfrac{(n^3p^{-1})^2}{(np)^{-2}}$

Simplify.

42. $3m^2 \bullet 2m$

43. $(a^3)^2$

44. $(-b^3c^4)^5$

45. $4m(3a^2m)$

Polynomials:

To add / subtract polynomials, combine like terms.

EX: $8x-3y+6-(6y+4x-9)$ *Distribute the negative through the parantheses.*

$=8x-3y+6-6y-4x+9$ *Combine terms with similar variables.*

$=8x-4x-3y-6y+6+9$

$=4x-9y+15$

Simplify.

46. $3x^3+9+7x^2-x^3$

47. $7m-6-(2m+5)$

To multiplying two binomials, use FOIL.

EX: $(3x-2)(x+4)$ *Multiply the first, outer, inner, then last terms.*

$=3x^2+12x-2x-8$ *Combine like terms.*

$=3x^2+10x-8$

Multiply.

48. $(3a+1)(a-2)$

49. $(s+3)(s-3)$

50. $(c-5)^2$

51. $(5x+7y)(5x-7y)$

Factoring.

Follow these steps in order to factor polynomials.

STEP 1: Look for a GCF in ALL of the terms.

a.) If you have one (other than 1) factor it out front.
b.) If you don't have one, move on to STEP 2.

STEP 2: How many terms does the polynomial have?

2 Terms

a.) Is it difference of two squares? $a^2 - b^2 = (a+b)(a-b)$

EX: $x^2 - 25 = (x+5)(x-5)$

b.) Is it sum or difference of two cubes? $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
$a^3 + b^3 = (a+b)(a^2-ab+b^2)$

EX: $m^3 + 64 = (m+4)(m^2 - 4m + 16)$
$p^3 - 125 = (p-5)(p^2 + 5p + 25)$

3 Terms

$x^2 + bx + c = (x+\quad)(x+\quad)$ **Ex:** $x^2 + 7x + 12 = (x+3)(x+4)$

$x^2 - bx + c = (x-\quad)(x-\quad)$ $x^2 - 5x + 4 = (x-1)(x-4)$

$x^2 + bx - c = (x-\quad)(x+\quad)$ $x^2 + 6x - 16 = (x-2)(x+8)$

$x^2 - bx - c = (x-\quad)(x+\quad)$ $x^2 - 2x - 24 = (x-6)(x+4)$

4 Terms -- Factor by Grouping

a.) Pair up first two terms and last two terms
b.) Factor out GCF of each pair of numbers.
c.) Factor out front the parentheses that the terms have in common.
d.) Put leftover terms in parentheses.

Ex: $x^3 + 3x^2 + 9x + 27 = (x^3 + 3x^2) + (9x + 27)$
$= x^2(x+3) + 9(x+3)$
$= (x+3)(x^2+9)$

Factor completely.

52. $z^2 + 4z - 12$

53. $6 - 5x - x^2$

54. $2k^2 + 2k - 60$

55. $-10b^4-15b^2$

56. $9c^2+30c+25$

57. $9n^2-4$

58. $27z^3-8$

59. $2mn-2mt+2sn-2st$

To solve quadratic equations, try to factor first and set each factor equal to zero. Solve for your variable. If the quadratic does NOT factor, use quadratic formula.

EX: $x^2-4x=21$ *Set equal to zero FIRST.*

$x^2-4x-21=0$ *Now factor.*

$(x+3)(x-7)=0$ *Set each factor equal to zero.*

$x+3=0 \quad x-7=0$ *Solve each for x.*

$x=-3 \quad x=7$

Solve each equation.

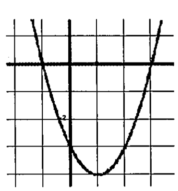
60. $x^2-4x-12=0$

61. $x^2+25=10x$

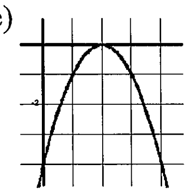
62. $x^2-14x+40=0$

DISCRIMINANT: The number under the radical in the quadratic formula (b^2-4ac) can tell you what kinds of roots you will have.

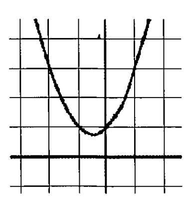
IF $b^2-4ac>0$ **you will have TWO real roots.**
(touches x-axis twice)



IF $b^2-4ac=0$ **you will have ONE real root**
(touches the x-axis once)



IF $b^2-4ac<0$ **you will have TWO imaginary roots.**
(Graph does not cross the x-axis)



QUADRATIC FORMULA – allows you to solve any quadratic for all its real and imaginary roots. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EX: In the equation: $x^2 + 2x + 3 = 0$, find the value of the discriminant, describe the nature of the roots, then solve.

$x^2 + 2x + 3 = 0$ Determine values for a, b, and c.

$a = 1 \quad b = 2 \quad c = 3$ Find dicriminant.

$D = 2^2 - 4 \cdot 1 \cdot 3$

$D = 4 - 12$

$D = -8$ *There are two imaginary roots.*

$$Solve: \quad x = \frac{-2 \pm \sqrt{-8}}{2}$$

$$x = \frac{-2 \pm 2i\sqrt{2}}{2}$$

$$x = -1 \pm i\sqrt{2}$$

Find the value of the discriminant, describe the nature of the roots, then solve each quadratic. Use EXACT values.

63. $x^2 - 9x + 14 = 0$

Discriminant = ________________

Type of Roots: ________________

Roots = ______________________

64. $5x^2 - 2x + 4 = 0$

Discriminant = ________________

Type of Roots: ________________

Roots = ______________________

Long Division – can be used when dividing any polynomials.
Synthetic Division – can ONLY be used when dividing a polynomial by a linear (degree one) polynomial.

EX: $\dfrac{2x^3+3x^2-6x+10}{x+3}$

Long Division

$\dfrac{2x^3+3x^2-6x+10}{x+3}$

$$
\begin{array}{r}
2x^2-3x+3+\dfrac{1}{x+3} \\
=x+3\overline{)\ 2x^3+3x^2-6x+10} \\
(-)\ \underline{(2x^3+6x^2)\qquad\qquad} \\
-3x^2-6x\qquad \\
(-)\ \underline{(-3x^2-9x)\qquad} \\
3x+10 \\
(-)\ \underline{(3x+9)} \\
1
\end{array}
$$

Synthetic Division

$\dfrac{2x^3+3x^2-6x+10}{x+3}$

$$
\begin{array}{r|rrrr}
-3 & 2 & 3 & -6 & 10 \\
 & \downarrow & -6 & 9 & -9 \\
\hline
 & 2 & -3 & 3 & 1
\end{array}
$$

$= 2x-3x+3+\dfrac{1}{x+3}$

Divide each polynomial using long division OR synthetic division.

65. $\dfrac{c^3-3c^2+18c-16}{c^2+3c-2}$

66. $\dfrac{x^4-2x^2-x+2}{x+2}$

To evaluate a function for a given value, simply plug the value into the function for x.

Evaluate each function for the given value.

67. $f(x)=x^2-6x+2$

$f(3)=$ ________

68. $g(x)=6x-7$

$g(x+h)=$ ________

69. $f(x)=3x^2-4$

$5[f(x+2)]=$ ________

Composition and Inverses of Functions:

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Suppose $f(x) = 2x,\ g(x) = 3x - 2,\ and\ h(x) = x^2 - 4$. **Find the following:**

70. $f\left[g(2)\right] =$ _______

71. $f\left[g(x)\right] =$ _______

72. $f\left[h(3)\right] =$ _______

73. $g\left[f(x)\right] =$ _______

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Example:

$f(x) = \sqrt[3]{x+1}$	Rewrite f(x) as y
$y = \sqrt[3]{x+1}$	Switch x and y
$x = \sqrt[3]{y+1}$	Solve for your new y
$(x)^3 = \left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3 = y + 1$	Simplify
$y = x^3 - 1$	Solve for y
$f^{-1}(x) = x^3 - 1$	Rewrite in inverse notation

Find the inverse, $f^{-1}(x)$ **, if possible.**

74. $f(x) = 5x + 2$

75. $f(x) = \frac{1}{2}x - \frac{1}{3}$

Rational Algebraic Expressions:

Multiplying and Dividing.

Factor numerator and denominator completely. Cancel any common factors in the top and bottom. If dividing, change divide to multiply and flip the second fraction.

EX:

$$\frac{x^2+10x+21}{5-4x-x^2} \bullet \frac{x^2+2x-15}{x^3+4x^2-21x}$$ *Factor everything completely.*

$$=\frac{(x+7)(x+3)}{(5+x)(1-x)} \bullet \frac{(x+5)(x-3)}{x(x-3)(x+7)}$$ *Cancel out common factors* in *the top and bottom.*

$$=\frac{(x+3)}{x(1-x)}$$ *Simplify.*

Simplify.

76. $\frac{5z^3+z^2-z}{3z}$

77. $\frac{m^2-25}{m^2+5m}$

78. $\frac{10r^5}{21s^2} \bullet \frac{3s}{5r^3}$

79. $\frac{a^2-5a+6}{a+4} \bullet \frac{3a+12}{a-2}$

80. $\frac{6d-9}{5d+1} \div \frac{6-13d+6d^2}{15d^2-7d-2}$

Addition and Subtraction.
First, find the least common denominator.
Write each fraction with the LCD.
Add / subtract numerators as indicated and leave the denominators as they are.

EX: $\frac{3x+1}{x^2+2x}+\frac{5x-4}{2x+4}$ *Factor denominator completely.*

$=\frac{3x+1}{x(x+2)}+\frac{5x-4}{2(x+2)}$ *Find LCD* $(2x)(x+2)$

$=\frac{2(3x+1)}{2x(x+2)}+\frac{x(5x-4)}{2x(x+2)}$ Re*write each fraction with the LCD as the denominator.*

$=\frac{6x+2+5x^2-4x}{2x(x+2)}$ *Write as one fraction.*

$=\frac{5x^2+2x+2}{2x(x+2)}$ *Combine like terms.*

81. $\frac{2x}{5}-\frac{x}{3}$

82. $\frac{b-a}{a^2b}+\frac{a+b}{ab^2}$

83. $\frac{2-a^2}{a^2+a}+\frac{3a+4}{3a+3}$

Complex Fractions.
Eliminate complex fractions by multiplying the numerator and denominator by the LCD of each of the small fractions. Then simplify as you did above

EX:

$$\frac{1+\frac{1}{a}}{\frac{2}{a^2}-1} \qquad \textit{Find LCD}: a^2$$

$$=\frac{\left(1+\frac{1}{a}\right)\bullet a^2}{\left(\frac{2}{a^2}-1\right)\bullet a^2} \qquad \textit{Multiply top and bottom by LCD.}$$

$$=\frac{a^2+a}{2-a^2} \qquad \textit{Factor and simplify if possible.}$$

$$=\frac{a(a+1)}{2-a^2}$$

84. $\dfrac{1-\frac{1}{2}}{2+\frac{1}{4}}$

85. $\dfrac{1+\frac{1}{z}}{z+1}$

86. $\dfrac{5+\frac{1}{m}-\frac{6}{m^2}}{\frac{2}{m}-\frac{2}{m^2}}$

87. $\dfrac{2+\frac{1}{x}-\frac{1}{x^2}}{1+\frac{4}{x}+\frac{3}{x^2}}$

Solving Rational Equations:

Multiply each term by the LCD of all the fractions. This should eliminate all of your fractions. Then solve the equation as usual.

EX:

$$\frac{5}{x+2}+\frac{1}{x}=\frac{5}{x} \qquad \textit{Find LCD first. } x(x+2)$$

$$x(x+2)\left(\frac{5}{x+2}\right)+x(x+2)\left(\frac{1}{x}\right)=\left(\frac{5}{x}\right)x(x+2) \qquad \textit{Multiply each term by the LCD.}$$

$$5x+1(x+2)=5(x+2) \qquad \textit{Simplify and solve.}$$

$$5x+x+2=5x+10$$

$$6x+2=5x+10$$

$$x=8 \quad \Leftarrow \textit{Check your answer. Sometimes they do not check!}$$

Check:

$$\frac{5}{8+2}+\frac{1}{8}=\frac{5}{8}$$

$$\frac{5}{10}+\frac{1}{8}=\frac{5}{8}$$

$$\frac{5}{8}=\frac{5}{8}$$

Solve each equation. Check your solutions.

88. $\frac{12}{x}+\frac{3}{4}=\frac{3}{2}$

89. $\frac{x+10}{x^2-2}=\frac{4}{x}$

90. $\frac{5}{x-5}=\frac{x}{x-5}-1$